Management Production and Inventories: From EOQ/EPQ to ELSP and its extension with shelf life and transitive demand items

WP8 Agri-food supply chain decision-making under uncertainty

Teaching Session

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Introduction

• At this session, the economic order quantity (EOQ) as one of the simplest and used models to control production and inventory is presented. After that, is presented Economic Lot Scheduling Problem which occurs when EPQ deals with more than one item. Some heuristics to implement ELSP approach are described. After that the particular case of a firm which package fresh vegetables is presented, describing the special characteristics of this framework regarding the management of the production and inventories.
Sections

• EOQ origins and framework
• EOQ vs EPQ
• EOP/EPQ to production and inventory management policies
• ELSP and solving methods
• Extensions and its implications on a tier 2 (agri-food supply chain)
EOQ origins

- In February 1913, Harris published in the “Factory, The Magazine of Management” a paper which open a wide research area.

- Paper starts:

   HOW MANY PARTS TO MAKE AT ONCE

   FORD W. HARRIS
   Production Engineer


   Interest on capital tied up in wages, material and overhead sets a maximum limit to the quantity of parts which can be profitably manufactured at one time; “set-up” costs on the job fix the minimum. Experience has shown one manager a way to determine the economical size of lots.

   Every manufacturer is confronted with the problem of finding the most economical quantity to manufacture in putting through an order. This is a general problem and admits of a general solution, and, however much it may be advisable to exercise judgment in a particular case, such exercise of judgment will be assisted by a knowledge of the general solution. Most managers, indeed, have a rather hazy idea as to just what this cost amounts to. If such is the case an investigation will show that the cost of handling, checking, indexing and superintending an order in the offices and shops is a considerable item and may, in a large factory, exceed one dollar per order.

   The setup cost proper is generally understood.
EOQ framework

• Production or inventory systems in which exists:
  – A demand rate for product (known and constant)
  – A cost (and a time) involved in releasing an order (administration, handling, setup..) \(\rightarrow\) decrease with large lots
  – A cost to storage the order \(\rightarrow\) decrease with small lot
  – An unit cost for acquire or produce an item

EOQ searches the balance between order/setup cost and holding inventory cost considering also cost for acquisition or manufacturing.
EOQ framework

• Assumptions:
  – A system which order or produce an item.
  – The demand rate $D$ for the product is known and constant.
  – Lead/Setup times ($ts$) and setup costs ($A$) are fixed and known.
  – The unit cost for the product ($c$) is known.
  – Holding costs $h$ are considered proportional to the units stored

EOQ searches the balance between order/setup cost and holding inventory cost considering also cost for acquisition or manufacturing.
EOQ formula

\[ TC(Q) = \frac{D}{Q} A + \frac{Q}{2} h \]

- \(D\) = annual demand for the item
- \(A\) = ordering costs for an order
- \(h\) = holding cost per unit per year
- \(TC\) = annual cost of ordering + annual cost of holding inventory

\[ \frac{dT C}{dQ} = - \frac{D}{Q^2} + \frac{h}{2} \rightarrow \min \frac{dT C}{dQ} = 0 \]

\[ Q = \sqrt{\frac{2AD}{h}} \]

* Usually is not considered because is independent of \(Q\) and their derivate is 0, so not affect EOQ formula.
EOQ/EPQ

Economic Order Quantity (EOQ):
- Harris (1913)
- Used when products are obtained from an outside supplier
  → Assumption Instantaneous replenishment

\[ Q = \sqrt{\frac{2AD}{H}} \]

Economic Production Quantity (EPQ):
- Taft (1918)
- Used when products are internally manufactured instead of being obtained from an outside supplier
  → Assumption that the order is received at a constant finite rate over time.
EPQ framework

• Assumptions:
  – A system which order or produce an item.
  – The demand rate $D$ for the product is known and constant.
  – Lead/Setup times ($ts$) and setup costs ($A$) are fixed and known.
  – The unit cost for the product ($c$) is known.
  – Holding costs $h$ are considered proportional to the units stored
  – The production rate ($p$) for the product is known and it is higher than its demand rate

EOQ searches the balance between setup cost and holding inventory cost considering also cost for adquisition or manufacturing.
EPQ formula

\[ TC(Q) = \frac{D}{Q} A + \frac{Q \left(1 - \frac{d}{p}\right) h}{2} \]

- \( D = \) annual demand for the item
- \( A = \) ordering costs for an order
- \( h = \) holding cost per unit per year
- \( p = \) production rate
- \( TC = \) annual cost of ordering + annual cost of holding inventory

\[ \frac{dTC}{dQ} = - \frac{D}{Q^2} + \frac{(1 - \frac{d}{p})h}{2} \rightarrow \min \frac{dTC}{dQ} = 0 \]

\[ Q = \sqrt[2]{\frac{2AD}{h \left(1 - \frac{d}{p}\right)}} \]

* Usually is not considered because is independent of \( Q \) and their derivate is 0, so not affect EOQ formula
EPQ / EOQ

$Q = \sqrt{\frac{2AD}{H}}$

$Q = \sqrt{\frac{2AD}{h \left(1 - \frac{d}{p}\right)}}$

d = demand for the item per unit time
p = production rate for the item per unit time
From EPQ to a Production and Inventory Management Policy

**EOQ /EPQ**

\[ Q = \sqrt{\frac{2AD}{H}} \]

\[ Q = \sqrt{\frac{2AD}{h(1 - \frac{d}{p})}} \]

- When produce/order?
- How much produce/order?
From EPQ to a Production and Inventory Management Policy

Questions determine by the production and inventory management policy:

• When produce/order?
• How much produce/order?
From EOQ to a Production and Inventory Management Policy

Questions determine by the production and inventory management policy:

• When produce/order?
  – setting a minimum stock level $s$ at which to reorder
  – establishing specific moments of time when to reorder ($T$)

• how much produce/order?

$s,T$ calculated using EOQ/EPQ

$Q = DT$
From EOQ to a Production and Inventory Management Policy

Questions determine by the production and inventory management policy:

• When produce/order?
• how much produce/order?
  – considering a stock level $S$ that serves as a reference to fill
  – fixed quantity $Q$ that optimizes the affected costs

$S$ calculated using EOQ/EPQ and $Q$ is EOQ/EPQ
From EOQ/EPQ to a Production and Inventory Management Policy

**EOQ /EPQ**

\[ Q = \sqrt{\frac{2AD}{H}} \quad Q = \sqrt{\frac{2AD}{h \left(1 - \frac{d}{p}\right)}} \]

**Production and Inventory Management Policy**

**Reorder point (s,Q):** order/produce Q when inventory level arrives s

**Model (s,S):** order/produce to S when inventory level arrives s

**Periodic review (T,S):** order/produce every T to S

**Model (T,Q):** order/produce every T the quantity Q
Deterministic / Stochastic Demand

• Assumptions:
  – A system which order or produce an item.
  – The demand rate $D$ for the product is known and constant.
  – Lead/Setup times ($ts$) and setup costs ($A$) are fixed and known.
  – The unit cost for the product ($c$) is known.
  – Holding costs ($h$) are considered proportional to the units stored
  – The production rate ($p$) for the product is known and it is higher than its demand rate.
Deterministic / Stochastic Demand

**Assumptions:**
- A system which order or produce an item.
- The demand rate for the product is stochastic stationary with mean $d$
- Lead/Setup times ($ts$) and setup costs ($A$) are fixed and known.
- The unit cost for the product ($c$) is known.
- Holding costs ($h$) are considered proportional to the units stored.
- The production rate ($p$) for the product is known and it is higher than its demand rate.
Multi-item EPQ: ELSP

- When more than one item has to be produced in a machine with finite capacity, starts a “fight” between items for the capacity of the machine, and a real sequencing problem starts.
- FEASIBILITY PROBLEM
The ELSP (Rogers, 1958; Bomberger, 1966; Madigan, 1968) programming several items $i=1..g$ on a single facility, where only one product can be produced at a time, with the objective of minimize the sum of holding costs and setup costs.

- The production rate per item, $p_i$ is deterministic and constant.
- Production setup times $A_i$ and setup costs $c_i$ are independent of the production sequence.
- Inventory holding costs $h_i$ are proportional to the inventory levels $I_i$.
- Product demand rates $d_i$ are deterministic or stochastic, but stationary – SELSP (stochastic lot scheduling problem)
- Production capacity is sufficient to meet demand, and all the demand has to be served.

SELSP (stochastic lot scheduling problem)
ELSP

- Objective: a programme in which several items are produced on the same facility on a repetitive basis.

  - Lotsizing:
    - Different Approaches to determine Qi (order quantity) or Ti (cycle length) for each item

  - Sequencing Problem:
    - Traditional Models with multi-items: (si, Si), (si, Qi), ...
    - RO-based (Segersted, 1999) heuristic to define the sequence
    - Other Heuristics

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\[ i, n \quad \text{Item indexes } i, n=1..g \]
\[ d_i \quad \text{Demand rate} \]
\[ p_i \quad \text{Production rate} \]
\[ A_i \quad \text{Setup Cost} \]
\[ h_i \quad \text{Holding Cost} \]
\[ c_i \quad \text{Setup time} \]
\[ T_i \quad \text{Cycle length} \]
\[ L_i \quad \text{Available inventory of item } i \]
\[ TC \quad \text{Total Cost} \]

NP-hard (Hsu, 1983)
ELSP - Lotsizing

- **EPQ, called Independent Solution – IS** (Harris, 1913)
  - cycles of independent manufacturing quantities

- **Common Cycle Approach** (Hanssmann, 1962)
  - Same cycle time for each product, enough bigger to fit one production of every product
    - if holding cost == setup cost

- **Basic Period Approach** (Bomberger, 1966)
  - Each product has its cycle time, but all the cycle time a multiple of a T period basic

- **Time Varying Lot size** (Maxwell, 1964)
  - Each product can be produced several times and with different batch sizes in a cyclic program

LOWER BOUND

\[ Q_i = d_i T_i \]

UPPER BOUND

\[ T = \sqrt{\frac{2 \sum_{i=1}^{n} A_i}{\sum_{i=1}^{n} h_i d_i \left(1 - \frac{d_i}{p_i}\right)}} \]

\[ Q_i = d_i T \]

heurisitical procedure for obtaining each \( T_i \)

(Doll and Whybark, 1973)

\[ Q_i = d_i T_i^e \]

\[ T_i^e = k_i T_{pb} \]

\[ Q_i = d_i T_i^v \]
ELSP – Scheduling: Basic Models

Reorder point \((s,Q)\) with \(ss^*\):
order/produce \(Q\) when inventory level arrives \(s\)

Model \((s,S)\) with \(ss^*\): order/produce to \(S\)
when inventory level arrives \(s\)

Periodic review \((T,S)\) with \(ss^*\):
order/produce every \(T\) to \(S\)

Model \((T,Q)\) with \(ss^*\): order/produce every \(T\) the quantity \(Q\)

Run Out: refers to the demand time units that are available in inventory.
* If the demand is stochastic

\[ s_i = ss_i + c_i d_i \]

\[ S_i = s_i + T_i d_i \frac{1 - d_i}{p_i} \]

\[ Q_i = d_i T_i \]

**EPQ**
Common Cycle Approach
Basic Period Approach
Time Varying Lot size

\[ RO_i = \frac{I_i}{d_i} \]

\[ RO_i = \frac{I_i - ss_i}{d_i} \]

\[ RO_i = \frac{I_i (1 - d_i/p_i)}{d_i} \]

\[ RO_i = \frac{I_i}{d_i} - c_i \]

\[ RO_1 < RO_2 < RO_3 < \ldots < RO_n \quad i,i',i'' \in \{1...n\} \]
ELSP – Extensions

<table>
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<tr>
<th>Variants about all the assumptions and more</th>
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The production rate per item, \( p \) is deterministic and constant.

| Production Rate stochastic
| Production Rate flexible
| Two products at the same time, coproduction |

Production setup times \( A_i \), and setup costs \( c_i \), are independent of the production sequence.

| Dependents Setups
| Unknown costs
| ...

Inventory holding costs \( h_i \), are proportional to the inventory levels \( I_i \).

| Deteriorating holding cost,
| Time variable holding costs…
| Unknown costs
| Items to order

Product demand rates \( d_i \) are deterministic or stochastic, but stationary – SELSP (stochastic lot scheduling problem)

| Demand stochastic stationary or not stationary |

Production capacity is sufficient to meet demand, and all the demand has to be served.

| Shortages: Back orders (total/partial), lost sales |

...
Extensions on ELSP according to a fresh vegetable producer

- Uncertainty on raw material
- Shelf Life, deteriorating items or perishable items
- Mix of product with substitutability and transitive
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